

Students' Conceptions of the Nature of Mathematics and Attitudes towards Mathematics Learning

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Abstract: Students' conceptions of the nature of mathematics have a great influence towards mathematics learning. In order to understand gender-wise students' conceptions and attitudes towards mathematics this study has been conducted in Karachi, Pakistan. This paper examines patterns in students' conception and attitude towards mathematics by analyzing survey data obtained from 82 students studying in grade eight in a private school context. The survey was conducted using a five-point Likert scale ranging from 'strongly agree' through 'neutral' to 'strongly disagree'. The survey findings illustrate that students consider mathematics as a useful subject which is used in daily life routines and facilitates in developing problem-solving skills and to strengthen future career. However, the findings also highlight students' confusion and contradictions in terms of the nature of mathematical knowledge i.e., they show their level of agreement to both 'absolutist' and 'fallibilist' view of mathematics. With respect to mathematics attitude the results shows that female students hold a more positive attitude towards mathematics and lesser mathematical anxiety than their male counterparts.

Keywords: conceptions, nature of mathematics, attitude, survey method, questionnaire

Introduction

Mathematics is at the heart of many successful careers and successful lives for societal development, particularly in the extraordinary and accelerating change circumstances. However, in reality, most people in general and students in particular dislike mathematics. The review of school-based educational research has revealed that the majority of secondary school students find mathematics as the most difficult, abstract, tedious, deadly, and boring subject (Ernest, 1996; Wong, Lam & Wong, 2001; Nardi & Steward, 2003; and Kirsti, Grevholm & Lepik, 2005). Moreover, research studies show that students in primary schools enjoy mathematics but when they move to secondary school their interest towards the subject declines (Lazim, Abu & Wan, 2003; Dossey,

Mullis, Lindquist & Chamber, 1988 cited in Ocak, 2006; Barber, 1994 cited in Chambers, 1998). The negative conceptions of mathematics have a major impact on students' achievement, enrollment in higher education and their future career decisions (Sam, 1999).

Generally, students' views of mathematics are developed based on their school learning experiences (Schoenfeld, 1989; Ernest, 1996) and how the public image of mathematics is portrayed in the society (Sam, 1999). To elaborate, in general it is believed that males are born with innate capabilities of making sense of abstract ideas and as mathematics is also an abstract level subject boys can do well as compared to girls (Walkerdine, 1998, Halai, 2006). Some of the other viewpoints students hold about mathematics include: mathematics problems have one and only one answer and they can be solved in a particular way; mathematics is a solitary activity, done by individuals in isolation; mathematics requires good memory and is only for clever ones.

In order to facilitate students to possess a positive image of mathematics there is a need to explore their existing conceptions and attitude towards the subject. Once we are aware of their conceptions then we can address their alternative conceptions through designing appropriate mathematics teaching and learning programmes. Therefore, this study is an attempt to make a contribution to the existing mathematics literature pertaining to children's learning of mathematics by systematic enquiry the following research question:

What conceptions do children hold about the nature of mathematics and attitude towards mathematics learning in a secondary school context in Karachi, Pakistan?

The terms 'conceptions' and 'beliefs' are used interchangeably in the research literature. As conceptions and beliefs are not directly observable and have to be inferred, therefore, it is problematic to have a common definition of these notions. Nevertheless, educationalists have tried to define these terms in a variety of ways. For instance, Pajares (1992) defines 'beliefs' as personal principles, constructed from experience that an individual employs, often unconsciously, to interpret new experiences and information and to guide action. Leatham (2006) refers to 'conceptions' as conscious or subconscious beliefs, understanding, meaning, mental images, and preferences. Based on these definitions, the working definition of these terms for the study is that 'conceptions are

conscious and unconscious cognitive and affective beliefs, personal meaning, mental images and preferences constructed from experiences within and beyond schooling.

Research Design

The main aim of this small scale quantitative study was to explore the general pattern of the students' conceptions of the nature of mathematics and their attitude towards mathematics learning. Therefore, a reasonable sample size of students (n=80) was identified to carry out the study. I opted for a survey design and used a directly administered questionnaire as the method for data collection. This seemed appropriate, as Kerlinger (1986) has noted, "survey research is probably best adapted to obtain personal and social facts, beliefs, and attitudes" (p.386). However, I was also aware of the limitations of survey methods. For instance, Kerlinger (1986) reminds us that "survey information ordinarily does not penetrate very deeply below the surface. The scope of the information sought is usually emphasized at the expense of depth" (p.387). In view of this limitation, I attempted to structure the questionnaire with a variety of questions on a five-point Likert Scale that tapped into the four areas that were the foci of the study.

Development of the Questionnaire

The process of developing the questionnaire began with a review of literature pertaining to the nature of mathematics and attitude and motivation towards learning mathematics. The two questionnaires, namely 'Modified Fennema-Sherman Attitude Scales' used by Kahveci and Imamoglu, 2006 and 'Attitude towards Mathematics Inventory questionnaire' used by Curtis, 2006 were also reviewed while developing the questionnaire. Consequently, the four major constructs - usefulness of mathematics; the nature of mathematics; attitude towards the subject as well as mathematics anxiety were identified for developing the questionnaire. Initially, forty items were designed and were given to experts for content validity.

Content Validity

The questionnaire was validated for its content by reviewers (n=6) with expertise in the field. Of these, half (n=3) had a mathematical background while the other half (n=3) were from a quantitative research background. Reviewers were asked to rate

items on a 5-point scale ranging from 1-not important to 5-very important. Descriptive reviewer feedback was also obtained. The criteria set for keeping the items in the final questionnaire were based on the experts' rating i.e. if any item is rated below three by 50% of the experts then it would be removed. Not a single item was rated below three. Based on the experts' rating some of the repetitive items (n=5) were removed while others were re-worded to make those simpler.

The content reviewers noted difficulties in words such as 'advancement', 'essence' and 'logic' and pointed out that these could have different meanings for different people. The items with those words were reworded. For example, 'the essence of school mathematics is to develop problem-solving skills of students' was re-written as 'mathematics is taught to develop problem-solving skills' in the final questionnaire. A suggestion was also offered to include a section on demographic information. Finally, the questionnaire consisting of 35 items was prepared to be pilot tested before being administered on the research sample.

Pilot Study

Pilot testing plays a key role in the context of surveys and is essential to ensure that the survey tool functions well before the final administration. Moreover, piloting needs to be done in a context which is similar to that of the main study. Therefore, one of the schools run by the same system was identified. The head teacher randomly selected students (n=12) of grade eight for the pilot study. First, the students were oriented about the study and were asked for their responses and comments to make the tool effective. Students were instructed to ask any clarification they needed on the items before responding to them. Most of the students asked for clarification regarding 'mathematics helps us to appreciate nature'. Interestingly, after students completed the questionnaire, we discussed ways to improve the items. The students' suggestions were quite helpful to improve the item. They suggested the item could be reworded as 'mathematics can be found in nature'. Finally, a short questionnaire with items (n=35) was designed to collect data on the students' conceptions of mathematics and their attitude towards mathematics learning. The rationale for designing a short and simple questionnaire was to provide a focus to the study and not to overwhelm respondents in terms of their time for completing

the questionnaires.

Research Sample

As this was a small scale study and the time available to conduct the study was limited, convenience sampling strategy was applied to identify the research sample. An English medium school which was easily accessible was identified as a research site. One of the reasons for selecting an English medium school was due to my own limitation of reading and writing Urdu. In this study, the questionnaire was administered to 82 students studying in grade eight. One of the reasons for selecting grade eight for the study was that grade eight is best 'representative' of a secondary level as this was half-way in the secondary school and I could explore the general tendency of secondary students' conceptions of the nature of mathematics and learning mathematics by engaging them in the study. Before administering the questionnaire the purpose and significance of the study was explained to the students and their willingness was sought for participating in the study. Overall, students (n=82) participated in the study. Table 1 represents the sample according to their age and gender. Most of the students were 13 to 15 years old.

Table 1: Demographic Information

Age group	Female	Male	Total
12	1	-	1
13	20	11	31
14	17	21	38
15	2	8	10
16	-	1	1
17	-	1	1
Total	40	42	82

Reliability of the Questionnaire

The internal consistency of the instrument was estimated using Cronbach-Alpha. Before running the Cronbach-Alpha all the five items of the sub-scale 'mathematical anxiety' were recoded. Generally, in social science research 0.6 is the accepted alpha value to estimate the internal consistency among the items in the tool (Antonius, 2003).

The result shows the overall internal consistency of the questionnaire is estimated at 0.813 which is high. A similar process was done to estimate the reliability coefficient on the four sub-scales (see table 2).

Table 2: Results of Cronbach-Alpha Estimates

Sub Scales	Number of items	Alpha Value
1. Mathematics value	10	0.623
2. Nature of mathematics	15	0.497
3. Attitude towards mathematics	5	0.827
4. Mathematics anxiety	5	0.688
Overall tool	35	0.813

The sub-scale ‘attitudes of mathematics’ (0.827) shows the highest alpha whereas ‘the nature of mathematics’ shows the lowest (0.497). The lowest alpha (0.497) is considered weak in terms of reliability; nevertheless, I found on the basis of literature review that nature of mathematics is a multidimensional concept and when Cronbach’s Alpha test is applied on multi-dimensional concepts the alpha value would be low (Sam, 1999). Therefore, I decided to retain this section even though the alpha value was low, as it gave valuable information regarding students’ conception of mathematics.

Data Analysis and Findings

In order to score the results each response was given a value. A value of five was given to a ‘strongly agree’ response, four was given to an ‘agree’ response, three was given to a ‘neutral’ response, two was given to a ‘disagree’ response and one was given to a ‘strongly disagree’ response. The frequency count, percentages, mean score and the standard deviation were calculated using SPSS. Reliability test, normality test, T test and Mann-Whitney test were performed while interpreting the data.

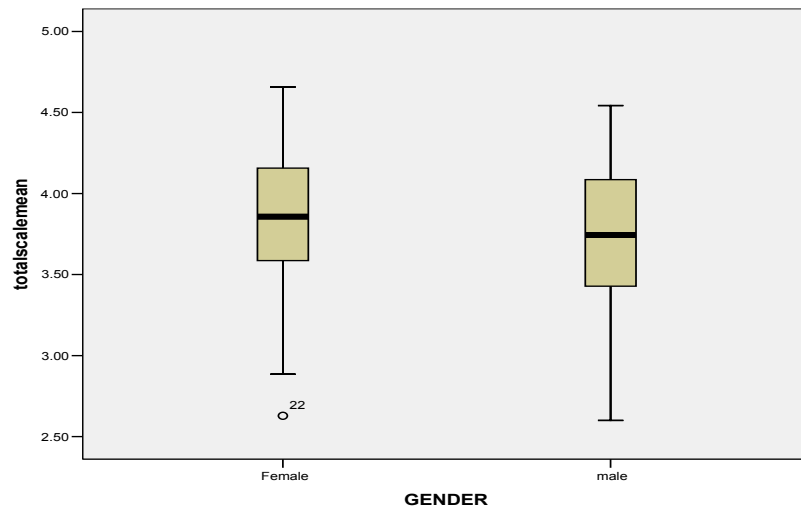
In this section, first of all the overall analysis of students’ responses to the questionnaire (n= 82; items= 35) is presented. Secondly, analysis based on the four sub-scales is presented. Individual items would be discussed with reference to gender to

elaborate the results. Finally, the implications of the study would be discussed.

Overall Analysis

The mean scores and the standard deviation of the all the 35 items is 3.8 and 0.5 with females scoring $M= 3.8$: $SD = 0.4$; and males scoring $M= 3.7$: $SD=0.5$, respectively. This shows that most of the students, irrespective of males and females, have agreed to most of the given statements which is evident by viewing the whisker plot (see Figure 1). Therefore, on the whole, not much difference between male and female responses was evident.

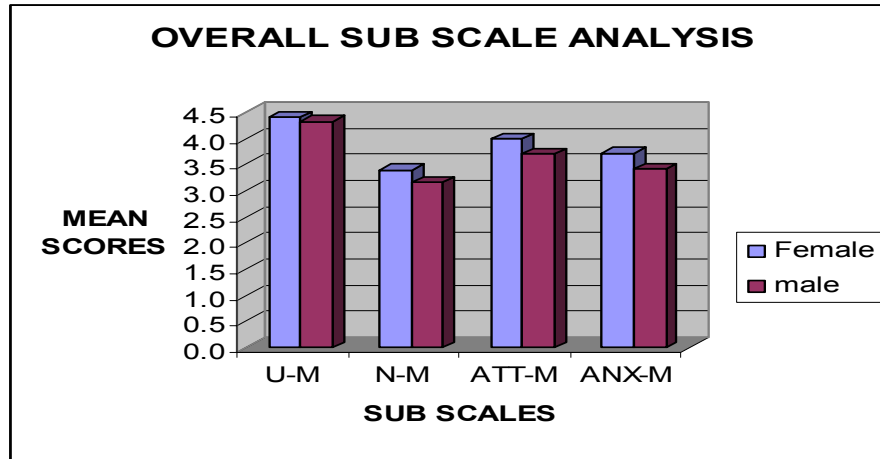
Figure 1: Gender wise Mean Score



Sub-scales wise Analysis

Table 3 shows that out of four subscales ‘usefulness of mathematics’ bears the highest mean score ($M= 4.4$, $n=82$) whereas the lowest is ‘nature of mathematics’ ($M=3.3$). This shows that most of the students agree to the items listed under this section and acknowledge that mathematics is a useful subject. In terms of ‘nature of mathematics’ students hold different views and therefore the mean score is low compared to other scales.

Figure 2



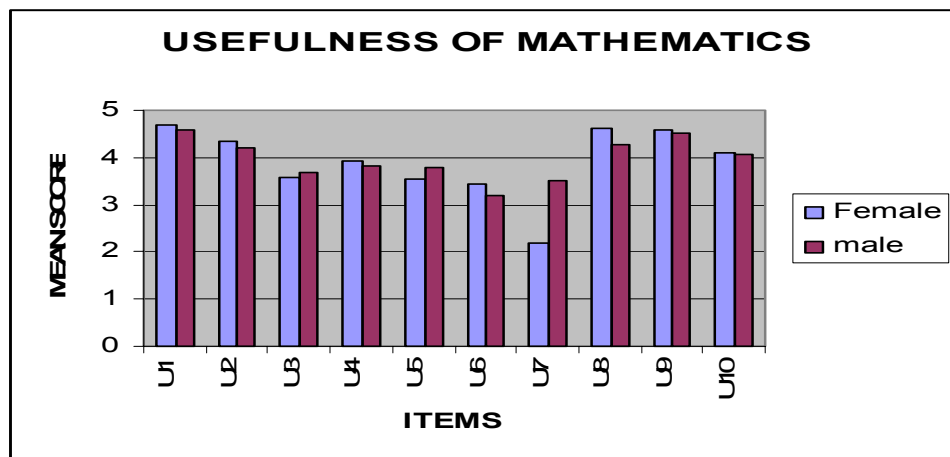
The mean score of the ‘attitude towards mathematics’ (M= 3.8) and ‘mathematics anxiety’ (M= 3.6) is quite similar. Very few students (13%) show a negative attitude towards mathematics and accept that they have anxiety of mathematics. Figure 2 shows that females’ mean score is little higher than male counterparts. Overall, there is no marked difference between them. Overall, the data shows that students consider mathematics as a useful subject as well as have positive attitude towards it. One of the possible reasons could be due to their positive experiences of learning mathematics in and beyond school. In contrast, most of large scale research studies done particularly in the secondary school context (Ernest, 1996; Wong, Lam and Wong, 2001; Nardi and Steward, 2003; and Kirsti, Grevholm & Lepik, 2005) show that mostly students dislike mathematics and have anxiety about the subject. Nevertheless, I am cautious that as this study is done on a small scale it cannot be generalized. In order to explore Pakistani students’ conceptions, a large scale study is required.

Usefulness of Mathematics

The overall rating for this subscale for females and males were 3.9 (SD =0.51) and 3.96 (SD = 0.57), respectively which is quite similar as well as high compared to other sub scales. Most of the students (93 %; 76 out of 82) agree that mathematics is used in daily life; it develops problem solving skills (91%); strengthens thinking processes (83%); and will help to get good jobs in future (87%).

The item-wise analysis shows some differences (figure 3) particularly in item 7 (i.e. ‘boys have more use of mathematics than girls do’) among female and male responses. Females scored lower (M= 2.2: SD=1.5) than males (M=3.5: SD=1.5). This means that more females than males disagree with this statement as they see the usage of mathematics equally for both males and females. This shows that male students do hold the view that mathematics is a masculine subject whereas female students consider mathematics as a subject for all. This was also identified by the level of significance test. The result showed that this was statistically significant (Mann-Whitney U= 442.500 with 82 cases; P= 0.01). Male students’ view towards mathematics as a masculine subject needs to be challenged in order to address their gendered view of mathematics.

Figure 3

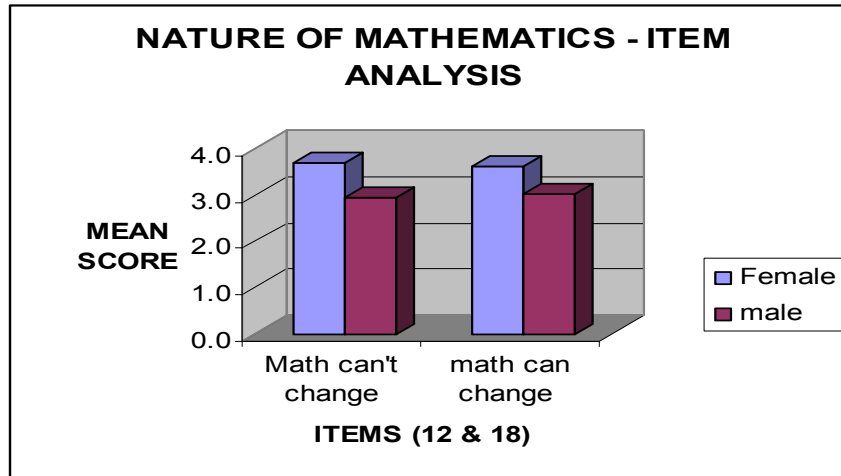


Nature of Mathematics

Both females and males mean score is relatively low compared to other subscales (females: M= 3.4: SD=1.1; males: M=3.2: SD = 1.1) but both females and males have relatively similar responses. However, further analysis highlighted students’ contradicting ideas with respect to the nature of mathematics.

Both male and female students’ show their level of agreement to both the items i.e., ‘mathematics cannot be changed’ and ‘current mathematics knowledge is open to future revision or change’ (see Figure 4).

Figure 4



This shows that students hold both an absolutist and fallibilist view towards mathematics at the same time. This view questions some of the models of the nature of mathematics suggested in the literature. For example, it is acknowledged that there exists two contrasting views of mathematical knowledge i.e. absolutism in one extreme and fallibalism at the other extreme (Lerman, 1990). Lerman further explained that mathematics within an absolutist paradigm is considered as an absolute, certain, infallible and objective body of knowledge. Proponents of this view of mathematics construct mathematics image as rigid, fixed, inhuman and pure where human experience has no place in creating mathematics. One right answer or method for mathematics problems could be the outcome of an absolutist view of the nature of mathematics. In contrast, “from the fallibilist perspective, mathematics is considered as fallible, and it is developed through conjectures, proofs, and refutations, and uncertainty is accepted as inherent in the discipline” (Thompson, 1992, p.132). One of the possible reasons for this contrasting view could be students’ own limited experience of learning mathematics and they might have experienced mathematics either as static or as changeable.

In addition, the data also illustrates a contradiction in considering mathematics as a problem solving subject as well as mathematics demands memorization. To elaborate, both female and male students’ mean score is high (between 4.5 and 4.6) with a standard deviation less than 1 for the items ‘mathematics is taught to develop problem

solving skills', 'mathematics learning strengthens our thinking' and 'there are many ways to solve a mathematics problem'. This shows that students' mathematical learning experience is positive which enabled them to view mathematics as a problem solving process and acknowledge that there are many ways to solve mathematical tasks. On the other hand, when students were asked to respond whether 'understanding the reason behind a formula is more important than simply memorizing it, the agreement level of both males and females was high (female = 78% and male = 79%). Also, when they were asked to respond whether 'mathematics can be learnt well mostly by memorizing rules and formula' again their agreement was quite high (female = 66% and male = 74%). This raises a question that why do students still consider memorizing the rules and formula when they acknowledge that reasoning is more important. This view needs to be further explored in order to understand students' experiences and views about mathematics.

Attitude towards Mathematics

Overall, students show a positive attitude towards mathematics ($M= 3.8$) and, therefore, their anxiety level is low. The data shows that more than 70% of the students enjoy learning mathematics and feel confident in doing mathematics. Moreover, 59% reported that they can learn mathematics easily. The reason for the positive attitude towards mathematics could be their positive learning experiences of viewing mathematics as a problem-solving process which strengthens their thinking. But there could be another contradictory interpretation similar to Pehkonen (2002) of the same finding that the students might have been engaged in very simple and easy mathematical tasks that lead them to consider mathematics as a very easy subject.

Figure 5A shows that in comparison to males, females have a more positive attitude towards mathematics. Moreover, Figure 5B shows that generally a female's level of anxiety is lower than male students. This shows a positive move as mathematics is increasingly being used in our technological world where professions are rarely attached to gender differences (Clifford, 1998).

Figure 5A

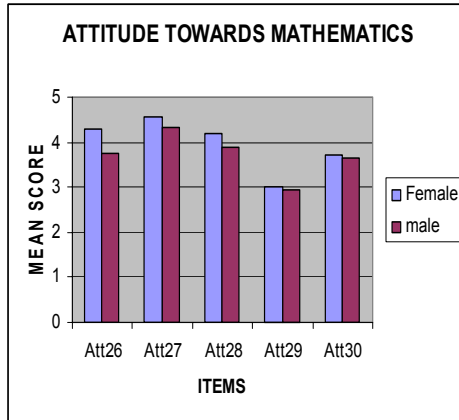
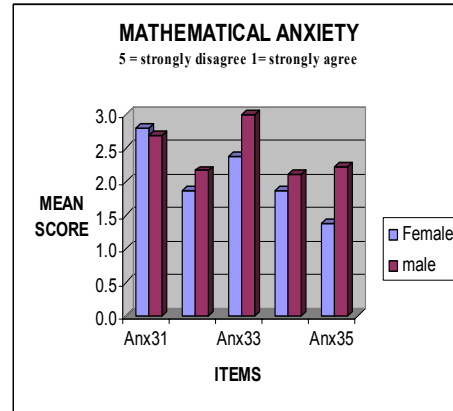


Figure 5B



Implication and Recommendation

In spite of the study being a small scale study some of the insights into students' conceptions of the nature of mathematics and their attitude towards learning mathematics have been explored. Interestingly, most of the students consider mathematics as an important subject which has relevance in their daily life. This idea needs to be sustained among students learning mathematics. Grouws, Howald and Calangelo (1996) assert that students' experiences, attitude, beliefs and conception of mathematics influence their learning of mathematics. Furthermore, Grootenboer (2002) asserts that if students' experiences of learning mathematics in school is positive then they develop a positive attitude towards mathematics and this supports in reducing mathematical anxiety. Therefore, in order to facilitate this view point, the mathematics curriculum needs to include topics and concepts that are relevant to children's daily life experiences. Also there is a need to engage students in problem-solving tasks so as to provide them with the experience that mathematics is useful in addressing issues and concerns that come across in the daily life.

Generally it is recognized that these mathematics curricula promote gender differences and explicitly demonstrate mathematics as a masculine subject. Walkerdine (1998) refers to Northam's (1983) study conducted in the developing world context

which analyzes mathematics text used in the classroom, shows that particularly in secondary schools most of the activities displayed are male oriented (football, cricket, and so on) those images determine maths as a masculine subject. Halai (2006) presents similar findings based on her analysis of the mathematics textbook prescribed by the Sindh textbook board. Her analysis shows that in the class VI textbook there are 51 word problems on the topic percentage, 96% of them refer to males through nouns and pronouns and only 4% refer to females. Therefore, we need to produce textbooks that promote equal importance to both females and males to learn mathematics.

In order to challenge the societal mathematics myths such as 'boys can learn mathematics well' or 'mathematics cannot be changed' one of the possible factors could be mathematics teachers who are aware of these myths and through their teaching practices they can convey that everyone can learn mathematics. Also can acknowledge that as mathematics is a human endeavor - it is a subject to be falsified and can be changed. In order to prepare teachers for this regard we need to include beliefs and conceptions of the nature of mathematics in teacher education programs.

Conclusion

This study highlights some interesting findings about students' conception of mathematics and their attitude towards mathematics. Both male and female students consider mathematics as a useful subject and possess positive attitude towards mathematics. However, their conception of the nature of mathematics shows both fallibilist as well as an absolutist view of mathematics.

In the end, I would like to acknowledge students' major contribution in this study and without them this study might not have been possible. I would also like to thank PhD course facilitators Dr. Anjum Halai, Dr. Philip Nagy and Dr. Meher Rizvi for their valuable comments and feedback on the study.

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